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E. Borchi and R. Gatto: MUON COUPLING PARAMETERS FROM RADIATIVE MUON ABSORPTION IN LIQUID HYDROGEN.

(Nota interna: n. 246)

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Radiative muon absorption has already been discussed by different authors(1). We have already examined(2) the possibility of detecting from observation of radiative muon absorption on nuclei possible terms in the weak axial current with second class bahaviour under G-transformation(3). In this note we discuss radiative muon absorption in liquid hydrogen including a possible term in the axial current with second class behaviour under G. The conserved vestor current hypothesis, recently confirmed by experiment(4), implies that the vector current has first-class behaviour under G. No experimental evidence exists however against second class axial contributions, usually discarded on the basis that they do not fit well in the current theoretical framework. Tests for the existence of such terms were proposed by Weinberg(3). Among such tests, the comparison of the mirror transitions B^{12}, $N^{12} \rightarrow C^{12}$ (with $J = 1$, no) indicates that the ft values for the two transitions differ by 14 ± 2.5 % or 16 ± 3 %(5). Such difference could be due to the possible second class term of the axial current. The matrix elements of the two mirror transitions could however differ also because of electromagnetic effects, which are difficult to estimate. The analysis by Huffaker and Greuling(6) is based on the assumption of equal matrix elements and indicates a large second class tensor contribution to the axial current together with a large pseudoscalar. The data on radiative muon absorption in Ca^{40}(7) do not agree well with the results of such analysis(2).

Muon absorption in liquid hydrogen is unfortunately of difficult interpretation because of molecular effects. Furthermore the very low radiative rate requires high intensity experiments, hard to perform at present. For the molecular problem we rely on Weinberg's analysis(8), hoping that a better determination of the parameters involved will be possible in the near future.

It is impossible at this time to derive reliable conclusion on a possible tensor contribution in the axial current from the recently performed experiments on μ-absorption in liquid hydrogen(9). We note however that the inclusion of such a coupling leads to a lower absorption rate, making the interpretation of the data easier. The measured muon capture rate in the hydrogen molecular ion seems in fact to be larger of about 20% than the theoretical prediction with the conventional choice of coupling constants (and with the most common choice $\xi = 1$ for Weinberg's parameter ξ). We find that inclusion of a tensor contribution in the axial current with a coupling constant $C_T \lesssim C_A$ (cases III and IV of Table I in the text) eliminates this discrepancy. A similar choice for C_T is also suggested from the interpretation of the CERN result on radiative muon capture in Ca^{40}(7) (see reference (2)). We would like to stress however that such suggestions do not yet offer definite evidence for the existence of C_T. In fact the liquid hydrogen data depend strongly on the rather unknown molecular mechanism and the interpretation of the Ca^{40} experiment may depend on the nuclear model used.

The calculations are made for the V-A theory with conserved vector current. The matrix elements of the vector current, V_λ, and of the axial-vector current, A_λ, are written as

$$V_\lambda = \bar{u}_n (C_V \gamma_\lambda - i C_M \sigma_{\lambda\mu} q_\mu) u_p \quad (1)$$

$$A_\lambda = \bar{u}_n (C_A \gamma_5 \gamma_\lambda + C_P \gamma_5 q_\lambda - i C_T \gamma_5 \sigma_{\lambda\mu} q_\mu) u_p \quad (1')$$

where: u_n and u_p are the neutron and proton spinors; $q_\lambda = (p-n)_\lambda$ with p and n denoting the neutron and proton momenta respectively; the form factors C depend on q^2 and are assumed real. The q^2 dependence is neglected in C_V, C_M, C_A and C_T whereas C_P is assumed to originate from one-pion exchange (see fig. 1) and is taken proportional to the pion propagator $(m^2 - q^2)^{-1}$ (10). The term proportional to C_T in (1') is usually dismissed on the basis that it has a different behaviour under $G = Ce^{i\pi I_2}$ from the other terms of A_λ(3). We discuss radiative muon absorption in hydrogen using exactly the same model that

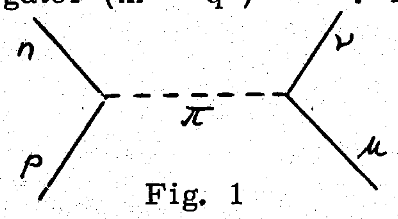

Fig. 1

we have used in discussing the radiative process in nuclei[2]. We derive the amplitude for $\mu^- + p \rightarrow n + \nu + \gamma$ under the assumptions that: (i) The terms proportional to C_A, C_T, C_V and C_M in the radiative amplitude originate from internal bremsstrahlung graphs (including the contributions from the anomalous moments); or from catastrophic graphs obtained through the substitution $q_\lambda \rightarrow q_\lambda - eA_\lambda$ in the terms proportional to C_A, C_T, C_V and C_M in the effective hamiltonian; (ii) The terms proportional to C_P are obtained by taking into account all the different possibilities of photon emission from the one-pion exchange graph of fig. 1 (The pion-lepton vertex is proportional to the virtual pion momentum and is assumed to give rise to an electromagnetic coupling through $q_\lambda \rightarrow q_\lambda - eA_\lambda$). A similar model is also used in the works by Manacher[11] and by Luyten, Rood and Tolhoek[12]. The coupling constants for muon absorption are taken as follows: $C_V = 0.97\, C_V^{(\beta)}$, $C_A = C_A^{(\beta)} = -1.25\, C_V$, $\mu C_M = 3.71\, C_V\, (\mu/m)$, $\mu C_P = 8\, C_A^{(\beta)}$, and (case I) $\mu C_T = 0$, (case II) $\mu C_T = 2(\mu/m) C_A$, (case III) $\mu C_T = 4\, (\mu/m) C_A$, (case IV) $\mu C_T = C_A$. We have called μ the muon mass and $C_V^{(\beta)}$ the vector β-decay constant. Calculation of the effective hamiltonian for $\mu^- + p \rightarrow n + \nu + \gamma$ in our model gives (for s-state absorption)

$$H_{\rm eff}^{(R,L)} = \frac{e}{(2\pi)^2} \frac{1}{\sqrt{2K}} \frac{1}{\pi a_o^3} (1 - \gamma_5^L)(\mathcal{E}_{R,L} + \gamma_5^N O_{R,L}) \tag{2}$$

where: K is the photon energy; a_o the muonic Bohr radius (with reduced mass); the upper indices L, N indicate wether the matrix acts on the nucleons or on the leptons; R and L are for emission of right-handed or left-handed circularly polarizefl photon; $\mathcal{E}_{R,L}$ and $O_{R,L}$ are given by:

$$\begin{aligned}
\mathcal{E}_{R,L} = & -\frac{1}{\mu}\left[C_V + \vec{\sigma}^L\cdot\vec{\sigma}^N C_A - i\,\vec{\sigma}^L \wedge \vec{\sigma}^N (\vec{\nu}+\vec{k})\, C_M + \vec{\sigma}^N (\vec{\nu}+\vec{k})\, C_T\right]\cdot \\
& \cdot \vec{\sigma}^L\cdot\vec{\epsilon}\, \frac{\lambda+1}{2} + i\,\vec{\sigma}^L \wedge \vec{\sigma}^N\cdot\vec{\epsilon}\, C_M - \vec{\sigma}^N\cdot\vec{\epsilon}\, C_T - \vec{\sigma}^L\cdot\vec{\epsilon}\, C_S \frac{1-\lambda}{2} + \\
& + \frac{1}{2m}\left[(\lambda - \vec{\sigma}^L\cdot\vec{\sigma}^N)(C_V - \lambda C_A) + (-i\lambda\,\vec{\sigma}^L \wedge \vec{\sigma}^N\cdot\vec{\nu} - K\,\vec{\sigma}^L\cdot\vec{\sigma}^N + \right. \\
& \left. + \vec{\sigma}^N\cdot\vec{\nu})(C_M + \lambda C_T) - \mu C_P^{(N)} - \lambda\mu C_S\right] \vec{\sigma}^N\cdot\vec{\epsilon} + \frac{\lambda}{2m}(\mu_p - \mu_n)\cdot \\
& \cdot\left[C_V\,\vec{\sigma}^N\cdot\vec{\epsilon} + C_A\,\vec{\sigma}^L\cdot\vec{\epsilon} - i\,C_M\,\vec{\sigma}^L \wedge \vec{\epsilon}\cdot\vec{\nu} + C_T(-K\,\vec{\sigma}^L\cdot\vec{\epsilon} + \vec{\nu}\cdot\vec{\epsilon}) + \right. \\
& \left. - \mu C_S\,\vec{\sigma}^N\cdot\vec{\epsilon}\right] + \frac{1}{2m}\frac{2\mu C_P^{(N)}}{(\vec{\nu}+\vec{k})^2 + \mu^2}(\vec{\epsilon}\cdot\vec{\nu})(\vec{\sigma}^N\cdot\vec{\nu})
\end{aligned} \tag{3}$$

$$\begin{aligned} O_{R,L} = & -\frac{1}{\mu}\left[\vec{\sigma}^L\cdot\vec{\sigma}^N C_V + C_A + \vec{\sigma}^N(\vec{\nu}+\vec{K})C_M - i\,\vec{\sigma}^L_\wedge\vec{\sigma}^N(\vec{\nu}+\vec{K})C_T\right]\cdot \\ & \cdot\vec{\sigma}^L\cdot\vec{\epsilon}\,\frac{\lambda+1}{2} + i\,\vec{\sigma}^L{}_\wedge\vec{\sigma}^N\cdot\vec{\epsilon}\,C_T - \vec{\sigma}^N\cdot\vec{\epsilon}\,C_M - \vec{\sigma}^L\cdot\vec{\epsilon}\,C_P^{(L)}\frac{1-\lambda}{2} + \\ & + \frac{1}{2m}\Big[(\lambda - \vec{\sigma}^L\cdot\vec{\sigma}^N)(-\lambda C_V + C_A) + (-i\,\vec{\sigma}^L{}_\wedge\vec{\sigma}^N\cdot\vec{\nu} - \lambda K\,\vec{\sigma}^L\cdot\vec{\sigma}^N + \\ & + \lambda\,\vec{\sigma}^N\cdot\vec{q})(C_M + \lambda C_T) - \lambda\mu\,C_P^{(N)} - \mu C_S\Big]\vec{\sigma}^N\vec{\epsilon} + \frac{\lambda}{2m}(\mu_p - \mu_n)\cdot \\ & \cdot\left[C_V\vec{\sigma}^L\cdot\vec{\epsilon} + C_A\vec{\sigma}^N\cdot\vec{\epsilon} + C_M(-K\vec{\sigma}^L\cdot\vec{\epsilon} + \vec{\nu}\cdot\vec{\epsilon}) - i\,C_T\vec{\sigma}^L_\wedge\vec{\epsilon}\cdot\vec{\nu} - \right. \\ & \left. -\mu\,C_P^{(N)}\vec{\sigma}^N\cdot\vec{\epsilon}\right] \end{aligned} \qquad (3')$$

In (3) and (3') we have also included possible contributions from a scalar term, of the form $C_S q_\lambda$, in the vector current V_λ. Such a term will however be ignored in the following, on the basis of the present evidence in favor of the conserved vector current hypothesis[4]. In (3) and (3') $\lambda = +1$ or -1 for R and L respectively; $C_R^{(N)}$ and $C_P^{(L)}$ are given by

$$C_P^{(L)} = C_P\,\frac{\mu^2 + m_\pi^2}{(\vec{\nu}+\vec{K})^2 + m_\pi^2} \quad ; \qquad C_P^{(N)} = C_P\,\frac{\mu^2 + m_\pi^2}{\vec{\nu}^2 - \vec{K}^2 + m_\pi^2} \qquad (4)$$

and $\vec{\nu}$ is the neutrino momentum.

We follow Weinberg's analysis of μ-capture[8] assuming that in liquid hydrogen the absorption(ordinary or radiative)occurs from the ortho-state of the muonic hydrogen molecular ion. The experimentally observed molecular absorption rate is given[8] by

$$\Lambda_{mol} = 2\gamma\left\{(\xi - \tfrac{1}{3})\Lambda_o + (\tfrac{4}{3} - \xi)\Lambda\right\} \qquad (5)$$

where Λ_o is the atomic absorption rate in hyperfine singlet state, and Λ is the absorption rate from unpolarized proton. According the Weinberg[8] $\gamma = 0{,}582$ and $1/2 \leq \xi \leq 1$. The rates Λ and Λ_o as calculated with the currents (1) and (1') in a non-relativistic approximation, are

$$\begin{aligned} \Lambda = & \frac{\mu^2}{\pi^2 a_0^3}\left(1 - \frac{2\mu}{m+\mu}\right)\Big[C_V^2 + 3C_A^2 + \mu^2 C_T^2 - 2\mu C_A C_T + 2\mu^2 C_M^2 - \\ & - 4\mu C_A C_M + \frac{\mu}{m}(C_V - C_A)^2 - \frac{\mu^2}{m}C_A C_P + \frac{\mu^3}{m}C_T C_P + \\ & + \frac{\mu^4}{4m^2}C_P^2 - \frac{\mu^3}{m^2}C_A C_P + 2\frac{\mu^2}{m}C_A C_T\Big] \end{aligned} \qquad (6)$$

$$\Lambda_0 = \frac{\mu^2}{\pi^2 a_0^3}\left(1-\frac{2\mu}{m+\mu}\right)\Big[C_V^2 - 6C_V C_A + 9C_A^2 + 4\mu^2 C_M^2 + \mu^2 C_T^2 + 4\mu^2 C_M C_T +$$
$$+ 4\mu C_V C_M + 2\mu C_V C_T - 12\mu C_M C_A - 6\mu C_A C_T + \frac{\mu}{m}(3C_V^2 - 12 C_V C_A +$$
$$+ 3C_A^2 + 5\mu C_V C_M + 5\mu C_A C_T + \mu C_V C_T + \mu C_A C_M - 9\mu^2 C_M C_T - 2\mu^2 C_T^2 + \tag{7}$$
$$+ \mu C_V C_P + 2\mu^2 C_M C_P - 3\mu C_A C_P + \mu^2 C_T C_P) + \frac{\mu^2}{2m^2}(3\mu C_V C_P - \mu C_A C_P +$$
$$+ 2\mu^2 C_T C_P + \frac{1}{2}\mu^2 C_P^2)\Big] .$$

These expressions are valid to order μ/m in $C_V^{(\beta)2}$. Relativistic corrections have been considered by Adams[13]. In table I we report, for the four choices considered for C_T, the values of Λ, Λ_o, Λ_o/Λ and of Λ_{mol} for $\xi = 1/2$ and $\xi = 1$.

TABLE I

Ordinary and radiative asorption rates in liquid hydrogen.

case	μC_T	$\Lambda / (\frac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_0^3})$	$\Lambda_o / (\frac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_o^3})$	Λ_o/Λ
I	0	4. 80	18. 50	3. 85
II	$2(\mu/m)C_A$	4. 62	16. 65	3. 62
III	$4(\mu/m)C_A$	4. 56	15	3. 30
IV	C_A	4. 95	11. 40	2. 30

case	$\Lambda_{mol}/(\frac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_0^3})$ $\xi = \frac{1}{2}$	$\Lambda_{mol}/(\frac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_0^3})$ $\xi = 1$	R_{mol} $(\xi = \frac{1}{2})$	R_{mol} $(\xi = 1)$
I	8. 22	16. 1	2. 55	0. 52
II	7. 75	14. 58	2. 94	0. 73
III	7. 34	12. 28	3. 40	0. 98
IV	7. 05	10. 65	4. 84	2. 12

A general feature to be noted is the decrease of both Λ and Λ_o with increase of C_T. We note that $\Lambda_o/\Lambda = 4$ would correspond to no absorption from the atomic hyperfine triplet state. Similarly $\Lambda_o/\Lambda = 1$ would correspond to equal absorption from the triplet and from the singlet. For small C_T the absorption is thus mostly from the singlet -- a well-know fact. For radiative absorption the situation is different: absorption from the singlet is small in the absence of C_T. A molecular absorption rate $\sim 450\ \mathrm{sec}^{-1}$ (9) is consistent with cases III and IV but would difficult to reconcile with case I. The photon spectrum from radiative absorption in muonic ortho hydrogen molecular ion can be written as

$$N_{mol}(x) = 2\gamma\left\{\left(\xi - \frac{1}{3}\right) N_o(x) + \left(\frac{4}{3} - \xi\right) N(x)\right\} \tag{8}$$

where $x = K/\mu$, and $N_o(x)$ and $N(x)$ are the spectra for singlet absorption and for absorption by an unpolarized photon. The spectra $N_o(x)$ and $N(x)$ can be evaluated from (2), (3) and (3') by averaging the squared matrix element over the singlet μ^--p state $\left(|\mathrm{singlet}\rangle = (2)^{-1/2}[\alpha^N\beta^L - \alpha^L\beta^N]\right)$ and on all four states respectively. The results for the ratio R_{mol} of radiative to ordinary absorption in the molecular ion are reported in Table I for two extreme choices of ξ: $\xi = 1/2$ and $\xi = 1$. The results depend strongly on the parameter ξ.

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